B.Tech Degree I & II Semester Examination in Marine Engineering, May 2008

MRE 101 ENGINEERING MATHEMATICS I

Time : 3 Hours | Maximum Marks : 100

(All questions carry ***EQUAL*** marks)

I. (a) In Lagrange's mean value theorem, find the value of θ if $f(x) = ax^2 + bx + c$ and the interval is $(0,1)$.

(b) Evaluate (i) $\underset{x \to 1}{Lt} \left(1-x^2\right)^{\frac{1}{Log(1-x)}}$

(ii) $\underset{x \to 0}{Lt} \left(\frac{1}{Sin\,x} - \frac{1}{x}\right)$

(c) If $y = e^{m Sin^{-1}x}$, prove that $\left(1-x^2\right)y_2 - xy_1 - m^2 y = 0$. Differentiate the above equation n times with respect to x, by using Leibnitz's theorem.

OR

II. (a) State Rolle's theorem and verify it for the function $f(x) = x(x+3)e^{-x/2}$ in $(-3,0)$.

(b) Evaluate (i) $\underset{x \to 0}{Lt} \left(\frac{a^x + b^x}{2}\right)^{1/x}$

(ii) $\underset{x \to 1}{Lt} \left[\frac{1}{Log\,x} - \left(\frac{x}{x-1}\right)\right]$.

(c) If $y = \left(x + \sqrt{1+x^2}\right)^n$ prove that $\left(1+x^2\right)y_{n+2} + x(2n+1)y_{n+1} + \left(n^2 - m^2\right)y_n = 0.$

III. (a) If $U = Log\left(\frac{x^5 + y^5}{x-y}\right)$, prove using Euler's theorem that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 4$.

(b) If $U = f(x-y, y-z, z-x)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

(c) Discuss the maxima and minima of $f(x,y) = xy^2(3x + 6y - 2)$

OR

IV. (a) If $U = \frac{xy}{x^2 + y^2}$, $x = Cos\,ht$, $y = Sin\,ht$, find $\frac{du}{dt}$, using chain rule.

(b) If u and v are functions of x and y and x, y are themselves functions of S and t then prove that $\left[\frac{\partial(u,v)}{\partial(x,y)}\right] \cdot \left[\frac{\partial(x,y)}{\partial(s,t)}\right] = \left[\frac{\partial(u,v)}{\partial(s,t)}\right]$.

(Turn Over)

(c) Find the percentage error in the area of an ellipse when an error of 1% each is made in measuring the major and minor axes.

V. (a) Find the vertex, focus and directrix of the parabola $y^2 - 2x - 6y + 5 = 0$.

(b) Find the equation of the hyperbola passing through $(2,3)$ and has the straight lines $4x + 3y - 7 = 0$ and $x - 2y - 1 = 0$ as asymptotes.

OR

VI. (a) Find the condition for the straight line $lx + my + n = 0$ to touch the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

(b) Show that the locus of mid-points of chords of the parabola $y^2 = 4ax$, which subtend a right angle at the vertex is $y^2 = 2a(x - 4a)$.

VII. (a) Find the area enclosed by one arch of the cycloid $x = a(\theta - Sin\theta), y = a(1 - Cos\theta)$ and its base.

(b) Evaluate (i) $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2)\,dxdy$.

(ii) $\int_{-1}^{1} \int_0^{z} \int_{x-z}^{x+z} (x + y + z)\,dxdydz$.

OR

VIII. (a) Using triple integral find the volume of the tetrahedron bounded by the Co-ordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

(b) Evaluate (i) $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dx\,dy}{1 + x^2 + y^2}$

(ii) $\int_0^1 \int_{y^2}^{1} \int_0^{1-x} xdz\,dx\,dy$.

IX. (a) If $\bar{a} + \bar{b} + \bar{c} = 0$ prove that $\bar{a} \times \bar{b} = \bar{b} \times \bar{c} = \bar{c} \times \bar{a}$. Hence deduce Sine formula of trigonometry.

(b) Find the values of the constants a, b, c for which the vector $\bar{f} = (x + y + az)i + (bx + 3y = z)j + (3x + cy + z)k$ is irrotational.

OR

X. (a) Prove that $\left[\bar{a} + \bar{b},\ \bar{b} + \bar{c},\ \bar{c} + \bar{a}\right] = 2\left[\bar{a}, \bar{b}, \bar{c}\right]$.

(b) If $\bar{r} = xi + yj + zk$, prove that div $\left(r^n \bar{r}\right) = (n + 3)r^n$.

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